

On the classification of spatially homogeneous 4D string backgrounds

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Abstract

A classification of all possible spatially homogeneous 4D string backgrounds (HSBs) has been obtained by appropriate ramification of the existing nine Bianchi types of homogeneous 3D spaces. A total of $24^2 = 576$ HSBs which have been classified as distinct contains a subclass of 192 which includes all possible FRW models as well as those in which $SO(3)$ isotropy is attained asymptotically. A discussion of these results also aims to facilitate the identification of HSBs which have already appeared in the literature. The basic physical perspective of the parameters of classification is outlined together with certain features relating to deeper aspects of string theory.

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1 Introduction

Considerable interest is being currently focused on spatially homogeneous [1] but not necessarily isotropic 4D string backgrounds [2]-[6]. The class of such spacetimes, hereafter recaled as HSBs, contains a large number of the known string backgrounds, which either descend from a conformal field theory and higher-dimensional compactifications, or simply satisfy the lowest-order string beta function eguations. At the limit of complete isotropy, taken mathematically or attained dynamically, the class of HSBs contains all possible Friedmann-Robertson-Walker or FRW-like backgrounds. On the other hand, non-isotropic HSBs seem to provide the best models for the understanding of anisotropy and its potentially crucial impact on the dynamics of the early universe [6], well before the attainment of the observed state of isotropy. This is precisely the region where the most fundamental cosmological problems arise and also where string theory seems to have its best chance of being confronted with reality. The class of HSBs admits a rather elegant mathematical treatment, exploitable for calculations as well as transparency to the deeper aspects of string theory.

It is not only desirable but also quite conceivable that most (if not all) HSBs will be found. With some progress in that direction already at hand [2]-[5], we may already proceed to establish a full classification scheme for all HSBs. By that we do not just mean the existing Bianchi classification of spatially homogeneous spacetimes, already exploited in the conventional formulation of general relativity [1]. The latter classification is of course fundamental but at the same time equivocally general, in the sense that spacetimes belonging to one and the same Bianchi type may differ profoundly in other important aspects. In other words, while the Bianchi assignments actually classify *types* of 3D homogeneous spaces (essentially their G_3 isometry groups with no regard to the 4D metric), we seek a classification of the 4D spacetimes themselves. With the first hints comming from the class of solutions presented in [6], it is by now evident that, in the context of string cosmology, the Bianchi-type assignments may be ramified in the above sense to yield a comprehensive and illuminating classification of all possible HSBs. With this paper we aim at presenting such a classification explicitly.

Of the total 576 HSBs classified, quite a few are presently known and we will try to fascilitate their identification in the existing literature. In the following section we have gathered some general definitions and facts needed for the presentation of our main results in section 3, further discussed in section 4. The latter also contains a tabulation of our brief review on the subclass of ‘diagonal’ HSBs. The quotation marks serve as a reminder that the diagonality of the metric

just been referred to holds in the invariant non-holonomic basis $\{\sigma^i\}$ (to be explicitly discussed shortly).

2 Preliminaries on the parameters of classification

We want to classify all 4D spacetimes with metrics of the form

$$ds^2 = -dt^2 + g_{ij}(t)\sigma^i\sigma^j, \quad (1)$$

as part of a background solution which satisfies at least the lowest-order string beta-function equations for conformal invariance. To fix notation and conventions used, we recall that these equations can be derived from the effective action [2]

$$S_{eff} = \int d^4x \sqrt{-g} e^\phi \left(R - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \partial_\mu \phi \partial^\mu \phi - \Lambda \right). \quad (2)$$

In this (so-called ‘sigma-’) conformal frame they are

$$R_{\mu\nu} - \frac{1}{4} H_{\mu\nu}^2 - \nabla_\mu \nabla_\nu \phi = 0, \quad (3)$$

$$\nabla^2(e^\phi H_{\mu\nu\lambda}) = 0, \quad (4)$$

$$-R + \frac{1}{12} H^2 + 2\nabla^2 \phi + (\partial_\mu \phi)^2 + \Lambda = 0, \quad (5)$$

where Λ is the cosmological constant emerging as a result of a non-vanishing central charge deficit in the original theory. In addition to the gravitational field $g_{\mu\nu}$, these expressions also involve the two other fundamental bosons present in the effective string action, namely the dilaton ϕ and, in the contractions $H_{\mu\nu}^2 = H_{\mu\kappa\lambda} H_\nu^{\kappa\lambda}$, $H^2 = H_{\mu\nu\lambda} H^{\mu\nu\lambda}$, the totally antisymmetric field strength $H_{\mu\nu\lambda}$. The latter, which may be equivalently viewed here as a closed 3-form, is defined in terms of the potential $B_{\mu\nu}$ as

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}. \quad (6)$$

The expression (1) adopted above gives in fact the most general *synchronous* metric possible. A universal time t is well defined in such manifolds, in which the $t = \text{const.}$ hypersurfaces of simultaneity actually coincide with the Σ^3 hypersurfaces of homogeneity, to be defined shortly. The metric coefficients, expressed in (1) in terms of the 3×3 matrix g_{ij} , can be functions of t only and $\{\sigma^i, i = 1, 2, 3\}$ is the already mentioned basis of 1-forms, invariant under the left action of a 3-parameter group of motions G_3 . In view of their fundamental importance, these

invariant forms will be precisely defined, together with the class of HSBs. The latter consists of all 4-dimensional spacetime manifolds M^4 which admit an r -parameter group of isometries G_r whose orbits in M^4 are 3-dimensional space-like hypersurfaces. As in conventional general relativity [1], these are precisely the hypersurfaces of homogeneity Σ^3 on which a G_3 subgroup of G_r acts transitively. In view of the dimensionality of M^4 , we can only have $3 \leq r \leq 6$. Any remaining symmetry (and the corresponding independent Killing vectors) must generate the $(r-3)$ -dimensional isotropy subgroup of G_r . Since there is no two-dimensional rotation group, $(r-3)$ can only have the values 0, 1, 3, the last one associated with the maximal FRW-type of symmetry. The action of G_3 is generally transitive on its orbits. There is only one exception, realized in the Kantowski-Sachs type of metric, in which G_3 is multiply transitive, actually acting on 2-dimensional space-like surfaces of maximal symmetry [1]. This is a marginal case (which is not to say it is unimportant!), which will be counted in our classification but it will not be considered any further in the present work. For the main and typical case, all possible isometry groups G_3 of the metric (1) are known and have been fully classified in nine Bianchi types, each identified by the corresponding set of group-structure constants C_{jk}^i . The latter also define (essentially uniquely) the invariant σ^i 1-forms by the relation

$$d\sigma^i = -\frac{1}{2}C_{jk}^i\sigma^j \wedge \sigma^k \quad (7)$$

Equivalently, the dual relation

$$[\xi_i, \xi_j] = -C_{ij}^k \xi_k, \quad (8)$$

involves a set of three independent Killing vectors ξ_i which form a basis dual to $\{\sigma^i\}$. All Bianchi-types have been further characterized in the literature [1] as being of class A if their adjoint representation is traceless, otherwise they are of class B.

Classification parameters are in principle expected to come from the cosmological constant Λ , the metric (1), as well as the dilaton and H fields. The last two must also respect the G_3 -isometries, namely their Lie derivatives wrt any Killing vector formed by the $\{\xi_i\}$ basis must vanish. Equivalently, the dilaton field must be a constant on Σ^3 and the H-field, when as a 3-form is projected in Σ^3 , must equal a constant times the invariant 3-form in Σ^3 . Thus, when viewed in M^4 , the dilaton ϕ can only be a function of the time t . On the other hand, the dual $\star H$ of H expressed in the same basis as (1) must be of the form

$$\star H = H_0^*(t)dt + H_i^*(t)\sigma^i. \quad (9)$$

namely with components H_μ^* at most functions of t in the $\{\sigma^i\}$ basis.

3 The classification of 4D HSBs

We are now in a position to proceed with a classification of all possible HSBs in 4D. The presence of a cosmological constant contributes with a factor of 2 in the number of possible HSBs, in other words we have two major subclasses corresponding the $\Lambda = 0$ and $\Lambda \neq 0$ cases. They will both be counted but all subsequent discussion will be restricted to the $\Lambda = 0$ case. To examine the rôle of the fundamental fields, we firstly observe that the dilaton, been a scalar and, in principle, an ever-present one, can hardly be expected to have any effect on the classification. To see explicitly that this is indeed the case, one must examine the contribution of the ϕ field in (3), which will be given shortly. We note, however that the result of this calculation is more illuminating if presented *not* in the frame in which (1) is expressed but rather in an orthonormal frame $\{\omega^\mu\}$. The latter may always be introduced (see, eg, [6]) so that (1) becomes

$$ds^2 = \eta_{\mu\nu} \omega^\mu \omega^\nu, \quad (10)$$

where $\eta_{\mu\nu}$ stands for the Minkowski signature $(-1, 1, 1, 1)$. In the so chosen $\{\omega^\mu\}$ basis and with a dot for d/dt we find

$$\nabla_\mu \nabla_\nu \phi = \begin{pmatrix} \ddot{\phi} & 0 \\ 0 & \gamma_{0ij} \dot{\phi} \end{pmatrix}, \quad (11)$$

where γ_{0ij} are the (non-holonomic) connection coefficients of (10). Considered as a 3×3 matrix, γ_{0ij} is diagonal whenever g_{ij} in (1) is diagonal. However, even in the latter case, the Ricci tensor can have non-vanishing off-diagonal components in the orthonormal frame. Thus the form of the contribution of the dilaton in the gravitational part of the field equations is always superceeded by the presence of purely gravitational contributions comming from g_{ij} .

Turning to the gravitational field, the rôle of g_{ij} is more transparent if examined in the same orthonormal frame, as one might have expected. We have just mentioned that even when diagonal, the g_{ij} matrix may give off-diagonal ($i \neq j$) equations in the set (3) in the $\{\omega_\mu\}$ frame. That happens in the case of Bianchi types which are of class B. However, in that case, the off-diagonal equations are just *constraints* on the diagonal ones [6]. In the case of a non-diagonal g_{ij} , obviously with up to 3 independent functions $g_{ij}(t)$, there will be as many independent off-diagonal equations. The rest of the off-diagonal equations in the set (3) will retain their constraint-like character. It is thus clear that there exist two major subclasses corresponding to g_{ij} been diagonal and non-diagonal. It should be recalled, however, that diagonality as well as dependence on the coordinates are frame-depended properties, so that the above picture would appear completely different if, instead of the *non-holonomic* frames

$\{\sigma^\mu\}$ and $\{\omega^\mu\}$, one employed conventional ones. We will not use the latter here but one could explicitly find holonomic-coordinate expressions by utilizing the appropriate entries supplied by the Table in the last section. Similar arguments and likewise dependence on t as well as on the spatial coordinates hold for any other G_3 -invariant quantity. This is in particular true for the components of H or its dual $\star H$, the contribution of which we will investigate next.

One can easily compute the contribution of H in (3) in the $\{\omega^\mu\}$ frame and subsequently express everything in terms of the dual $\star H$. As a result of this straightforward calculation one distinguishes four categories of H -field configurations which we may denote in order of increasing complexity as $(0), (\uparrow), (\rightarrow), (\nearrow)$. Of these, the first one corresponds to the trivial case of vanishing H . Next is the case of H fields such that their dual $\star H$ (seen as a vector) is orthogonal to the (pictured as ‘horizontal’) hypersurfaces of homogeneity Σ^3 . Equivalently, a (\uparrow) configuration is such that $H_i^* = 0$ in (9). Complementary to that is the ‘transverse’ case of a $\star H$ lying entirely within the Σ^3 hypersurfaces, namely with $H_0^* = 0$, suggestively denoted as (\rightarrow) . H fields ‘tilted’ between the two extremes correspond to the general (9) case, represented by the tilted arrow (\nearrow) . Of all four configurations only the first two have been deployed in the literature on HSBs until now. They will all be further discussed in the last section.

The classification of HSBs may now proceed as follows. Without the need to any explicit reference to the values of Λ and ϕ , each classified HSB may be generically codified as $X(n, d, a)$. In that acronym, X will generally denote the Bianchi type of G_3 , so that X essentially takes the values I, II, \dots, IX . It should be noted, however, that the multitude of the distinct values taken by X is in fact 12: nine for the conventional Bianchi types, plus two for a necessary refinement (concerning types VI_h and VII_h , as it will be explicitly seen in the sequel) plus one for the Kantowski-Sachs case. The argument n specifies the isotropy group and takes three distinct values: it equals 3 for $SO(3)$ (the case of complete isotropy), it equals 2 for $SO(2)$ (only two principal directions are equivalent) and it is omitted altogether if there is no isotropy group. The next argument takes two distinct values: it will be present only when the g_{ij} matrix in (1) is diagonal. The last argument (a) takes the four values $(0), (\uparrow), (\rightarrow), (\nearrow)$, just discussed. Thus the product of the multiplicities of the three arguments is $3 \times 2 \times 4 = 24$. Taking also into account the multiplicity of X , as well as the factor of 2 coming from the cosmological constant as mentioned, we arrive at the earlier quoted number of $24^2 = 576$ HSBs, including a subclass of FRW or FRW-like models which will be counted in the next section. In the rest of this section we will briefly discuss the $X(d)$ backgrounds, namely the subclass of ‘diagonal’ metrics. As mentioned, an effort will be made to facilitate the identification of HSBs which are already

known, however without aiming at what should rather be the objective of a review article. These results will be further discussed in the last section.

Type I. Spacetimes of this type admit an isotropy limit which contains all possible flat ($k = 0$) FRW-type of HSBs. Until recently, only the $I(3d0)$ model and its generalizations $I(3d \uparrow)$ and the Kasner-like $I(d0)$ had been given [3],[4],[5]. They are all reproduced as special cases of the $I(d \uparrow)$ HSB given in [6]. Very recently, the $I(d \rightarrow)$ has been found, together with the result that there exist no $I(d \nearrow)$ solution [7].

Type II. The fully anisotropic $II(d \uparrow)$ was given in [6], generalizing the $II(d0)$ found in [5]. For the $II(d \rightarrow)$ and $II(d \nearrow)$ cases the same hold as for type I.

Type III. The general anisotropic solution, namely $III(d \nearrow)$, exists and reduces to the $III(d \uparrow)$ found in [6] which in turn reduces to the $III(d0)$ found in [5]. However, there also exists a general $III(d \rightarrow)$ HSB which cannot be reached as a limit of the mentioned $III(d \nearrow)$ (cf. [7]).

Type IV. There exist no $IV(d)$ HSBs, as all diagonal solutions are singular everywhere [6],[7].

Type V. For HSBs of this type the isotropy limit attainable gives rise to the open ($k = -1$) FRW-type spacetimes. Such are the $V(3d0)$ and $V(3d \uparrow)$ cases found in [3], [4], respectively. The first has been generalized by the $V(d0)$ in [5], while all are special cases of $V(d \uparrow)$ HSB found in [6]. The later is in turn the obvious limit of the general ‘diagonal’ case, namely $V(d \nearrow)$. Curiously enough, there exists no $V(d \rightarrow)$ solution [7].

Type VI_h . The G_3 involved in this type of models actually form a continuous 1-parameter family of groups parametrized by h , with the values $h \neq 0, 1$ typically excluded as giving rise to Bianchi types III and V respectively (all these types are of class B). The results of the previous case are generally valid here as well, except of course for the isotropy limit. The real exception is with the $h = -1$ case, which must be and is discussed separately next.

Type VI_{-1} . HSBs in this case are generally *not* obtained at the $h = -1$ limit from solutions of the previous type. The general ‘diagonal’ solution, namely $VI_{-1}(d \nearrow)$ has been recently found. It generalizes the $VI_{-1}(d \uparrow)$ in [6], and also gives a non-trivial $VI_{-1}(d \rightarrow)$ limit [7].

Type VII_h . In this case, which also involves a 1-parameter group G_3 (here with $h^2 \leq 4$), all solutions are singular everywhere, namely there exist no $VII_h(d)$ HSBs unless $h = 0$. The latter case (which exceptionally involves space-times of class A) must be and is considered

separately next.

Type VII_0 . There exists the expected $VII_0(d \uparrow)$ with isotropy limits $VII_0(2d \uparrow)$ and $VII_0(3d \uparrow)$ as in the Type-I case [6]. As recently found, there also exists the $VII_0(d \rightarrow)$ but no $VII_0(d \nearrow)$ solution [7].

Type VIII. The $VIII(2d \uparrow)$ has been found in [6], while neither $VIII(d \nearrow)$ nor $VIII(d \rightarrow)$ exist [7].

Type IX. The $IX(2d \uparrow)$ case (namely the analogue of the well-known Taub metric to which it reduces) has been recently found [6]. The complete isotropy limit, namely $IX(3d \uparrow)$, exists and reproduces the closed ($k = 1$) FRW-type of solutions found in [3],[4]. There are no other ‘diagonal’ Bianchi-type IX HSBs [7] except for the elusive $IX(d \uparrow)$ (escaping us just like its Mixmaster counterpart!).

4 Conclusions

A classification of all possible 4D HSBs, codified as $X(n,d,a)$, has been presented. It is based on the known Bianchi classification of the isometry groups G_3 which generate all homogeneous 3D spaces, so that X takes the values I,II,...,IX etc (roughly one for each of the Bianchi types plus one for the Katowski-Sachs class of metrics). Of the the arguments n,d,a one or more may be omitted or take values as follows. The integer n , which identifies the isotropy group, takes the value 3 for $SO(3)$, 2 for $SO(2)$ and it is omitted in the case of complete anisotropy. The argument d is omitted only when g_{ij} in (1) is *non*-diagonal in the synchronous frame of the invariant $\{\sigma^i\}$ basis. The third argument, zero in the trivial case of an identically vanishing H field, indicates the orientation of $\star H$ wrt the hypersurfaces of homogeneity Σ^3 .

The quoted number of $24^2 = 576$ HSBs which the classification sees as distinct, includes a subclass of 192 backgrounds which are either FRW or could asymptotically attain full $SO(3)$ isometry. They can be easily counted because they may only descend from type I(and VII) for the flat ($k=0$) case, or from type V for the open ($k=-1$), or from type IX for the closed ($k=1$) models (so their multitude is $2 \times 4 \times (3 \times 2 \times 4)$). Some of the HSBs which the classification sees as distinct are in fact special cases of higher symmetry, while others cannot be considered as practically realizable because their manifolds are singular everywhere. Such is the case for those denoted by \nearrow in the the Table below. The latter gives a summary of the ‘diagonal’ types discussed in the previous section, with the arrow \Rightarrow pointing towards special sub-cases, mostly the just-mentioned higher-symmetry limits.

The most rudimentary physical aspects of the parameters of classification have already been discussed, particularly concerning the isometry and isotropy groups. Remaining to be so discussed is the parameter which specifies the orientation of any given H-field configuration. That orientation may be better visualized in terms of an associated *congruence* of flow lines to which $\star H$ is tangent. Such a congruence will cross the family of Σ^3 (as the latter evolves in time) orthogonally (\uparrow), or it may be tilted (\nearrow), possibly to the limit of lying entirely within those hypersurfaces (\rightarrow). It will suffice to examine the Bianchi-type V case, for which we copy from our Table the sequence

$$\cdots V(d \nearrow) \Rightarrow V(d \uparrow) \Rightarrow \cdots \Rightarrow V(3d \uparrow). \quad (12)$$

In the $V(d \nearrow)$ model the $\star H$ congruence will have general kinematics, namely it will exhibit expansion, shear (anisotropy) and, in principle, vorticity as well. In the next more symmetric case $V(d \uparrow)$ vorticity cannot be sustained, while in the fully symmetric $V(3d \uparrow)$ (namely the open FRW model) only expansion has survived.

We have already mentioned that, in the context of HSBs classified here, one might appropriately examine certain deeper aspects of string theory. These are mostly related to the construction of realistic CFTs or the retention of conformal invariance and other symmetries under duality transformations (cf, eg, [5],[8] and refs cited therein). Here we will briefly discuss the behavior of HSBs under abelian target space duality. As first noted in [6], $X(d \uparrow)$ backgrounds produce duals with metrics in the same class, except for the cases marked as \exists^* in the table. These, apparently losing most of their symmetry, give rise to non-homogeneous backgrounds. However, even the relatively much simpler $X(d \rightarrow)$ class may produce highly non-trivial duals (inhomogeneous and without universal time in view of the emergence of g_{ti} components). For the general case, it appears that $X(\nearrow)$ may generate backgrounds so general that they would be virtually unreachable by any other approach.

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Classification of ‘diagonal’ 4D HSBs

Coordinate basis	X	$d \rightarrow$	$d \nearrow$	$d \uparrow$	$2d \uparrow$	$3d \uparrow$
$\sigma^i = dx^i$	I	\exists	\nexists	$\exists \Rightarrow$	$\exists \Rightarrow$	\exists
$\sigma^1 = dx^2 - x^1 dx^3$ $\sigma^2 = dx^3$ $\sigma^3 = dx^1$	II	\exists	\nexists	\exists	\nexists	\nexists
$\sigma^1 = dx^1$ $\sigma^2 = dx^2$ $\sigma^3 = e^{x^1} dx^3$	III	\exists	$\exists \Rightarrow$	\exists	\nexists	\nexists
$\sigma^1 = e^{-x^1} dx^2 - x^1 e^{-x^1} dx^3$ $\sigma^2 = e^{-x^1} dx^3$ $\sigma^3 = dx^1$	IV	\nexists	\nexists	\nexists	\nexists	\nexists
$\sigma^1 = dx^1$ $\sigma^2 = e^{x^1} dx^2$ $\sigma^3 = e^{x^1} dx^3$	V	\nexists	$\exists \Rightarrow$	$\exists \Rightarrow$	$\exists \Rightarrow$	\exists
$\sigma^1 = dx^1$ $\sigma^2 = e^{hx^1} dx^2$ $\sigma^3 = e^{x^1} dx^3$	VI_h	\nexists	$\exists \Rightarrow$	\exists	\nexists	\nexists
$(h = -1)$	VI_{-1}	\exists	$\Leftarrow \exists \Rightarrow$	\exists	\nexists	\nexists
$\sigma^1 = (A - kB)dx^2 - Bdx^3$ $\sigma^2 = Bdx^2 + (A + kB)dx^3$ $\sigma^3 = dx^1$	VII_h	\nexists	\nexists	\nexists	\nexists	\nexists
$(h = 0)$	VII_0	\exists^*	\nexists	$\exists^* \Rightarrow$	$\exists \Rightarrow$	\exists
$\sigma^1 = dx^1 + ((x^1)^2 - 1)dx^2 + Z_1 dx^3$ $\sigma^2 = dx^1 + ((x^1)^2 + 1)dx^2 + Z_2 dx^3$ $\sigma^3 = 2x^1 dx^2 + Z_3 dx^3$	$VIII$	\nexists	\nexists	$\exists^* \Rightarrow$	\exists	\nexists
$\sigma^1 = -\sin x^3 dx^1 + \sin x^1 \cos x^3 dx^2$ $\sigma^2 = \cos x^3 dx^1 + \sin x^1 \sin x^3 dx^2$ $\sigma^3 = \cos x^1 dx^2 + dx^3$	IX	\nexists	\nexists	$\exists^* \Rightarrow$	$\exists \Rightarrow$	\exists

notes:

\exists^* indicates that the solution exists but it has not been found in closed form.

The last column gives the FRW limits.

$$A = e^{-kx^1} \cos(qx^1), \quad B = -\frac{1}{q} e^{-kx^1} \sin(qx^1), \quad [k = \frac{h}{2}, q = \sqrt{1 - k^2}]$$

$$Z_1 = x^1 + x^2 - x^2(x^1)^2, \quad Z_2 = x^1 - x^2 - x^2(x^1)^2, \quad Z_3 = 1 - 2x^1 x^2.$$

References

- [1] G.F.R. Ellis and M.A.H. MacCallum, Commun. Math Phys. 12 (1969)108; 19 (1970)31.
M. Ryan and L. Shepley, *“Homogeneous Relativistic Cosmologies*, Princeton Univ. Press, Princeton, 1975;
M.A.H. MacCallum, *“Anisotropic and Inhomogeneous Relativistic Cosmologies”*, in *“General Relativity-An Einstein Centenary Survey”*, eds. S. W. Hawking and W. Israel, Cambridge Univ. Press, Cambridge, 1979;
D. Kramer *et. al.*, *“Exact solutions of Einstein’s field equations”*, Cambridge Univ. Press, Cambridge, 1980;
E.W. Kolb and M.S. Turner, *“The Early Universe: Reprints”*, Addison-Wisley, N.Y., 1988;
“The Early Universe”, Addison-Wesley, N.Y., 1990.
- [2] E. Fradkin and A. Tseytlin, Nucl. Phys. B261 (1985)1;
C. Callan, D. Friedan, E. Martinec and M. Perry, Nucl.Phys. B262 (1985)593.
- [3] R. Myers, Phys. Lett. B199 (1987)371;
R. Brandenberger and C. Vafa, Nucl. Phys. B316 (1988)319;
I. Antoniadis, C. Bachas, J. Ellis and D.V. Nanopoulos, Phys. Lett. B211 (1988)393 ; Nucl. Phys. B328 (1989)117;
M. Muller, Nucl. Phys. B337 (1990)37;
K.A. Meissner and G. Veneziano, Phys. Lett. B267(1991)33;
A. Sen, Phys. Lett. B271 (1991)295;
M.Gasperini, J. Maharana and G. Veneziano, Phys. Lett. B272 (1991)277;
A.A. Tseytlin, Class Quantum Grav. 9 (1992)979;
A.A. Tseytlin and C. Vafa, Nucl. Phys. B372 (1992)443;
C. Kounnas and D. Lüst, Phys. Lett. B 289 (1992)56;
M. Gasperini and G. Veneziano, Phys. Lett. B277 (1992)256; Mod. Phys. Lett. A8 (1993)3701;
M. Gasperini, R. Ricci and G. Veneziano, Phys. Lett. B319 (1993)438.
- [4] E.J. Copeland, A. Lahiri and D. Wands, Phys. Rev. D50 (1994)4868 ; *“String cosmology with a time-dependent antisymmetric-tensor potential”* preprint Sussex-Ast/94/10-1, hep/th/9410136.

- [5] M. Gasperini and R. Ricci, “*Homogeneous conformal string backgrounds*”, preprint DFTT-02/95, hep-th/9501055.
- [6] N.A. Batakis and A.A. Kehagias, “*Anisotropic space-times in homogeneous string comology*”, preprint CERN-TH/95-14, hep-th/9502007.
- [7] N.A. Batakis, “*A new class of homogeneous string backgrounds*”, preprint (to appear) CERN-TH/95-000, hep-th/9500000.
- [8] E. Kiritsis and C. Kounnas, Phys. Lett. B331 (1994) 51;
 E.Alvarez, L. Alvarez-Gaumé and Y. Lozano, “*On non-abelian duality*”, Preprint CERN-TH. 7204/94;
 I. Bakas and K. Sfetsos, “*T-duality and world-sheet supersymmetry*”, preprint CERN-TH/95-16, hep-th/9502065.